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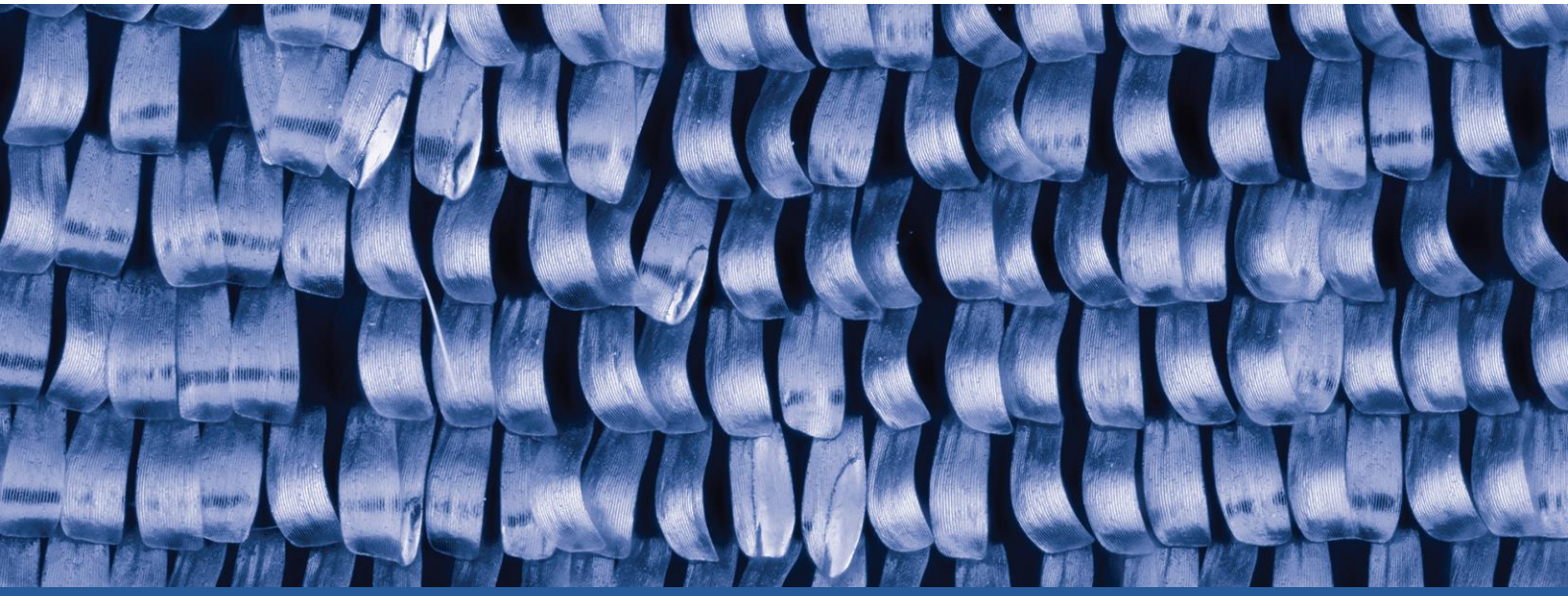

PC National model and database: the basic version

Technical paper

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PC National model and database: the basic version

PC National is a computable general equilibrium (CGE) model of the Australian economy that has been used in some recent Productivity Commission reports: *Rising Protectionism* (PC 2017), *The Nuisance Cost of Tariffs* (PC 2022) and *National Competition Policy* (PC 2024).

The model consists of a basic version and various application-specific versions.

The basic version is designed to be used as a simple structured model that is easily adaptable for any policy application. As a result, it contains only the core variables and equations required to solve for the model's general equilibrium solution, plus a few non-core variables commonly used for aggregating and reporting simulation results (such as major national indicators). It does not include any modifications and enhancements made for specific projects. The database for the basic version is based only on information from ABS national accounts, particularly the official Australian input-output tables (ABS 2021).

Application-specific versions of PC National include additional policy and reporting variables tailored to the application. The inclusion of this additional detail should not affect the model solution. Recent application-specific examples include *Rising Protectionism* (PC 2017) and *National Competition Policy* (PC 2024).

This paper documents the basic version of the PC National model and database. The paper has five sections. Section 1 provides an overview of PC National. Section 2 discusses the construction of the database. Section 3 outlines the model's theory and the assumptions that underlie the core equation system. Section 4 discusses macro closure options for policy simulations and their underlying assumptions. Section 5 defines key national indicators used to report on aggregates.

1. Overview of PC National

PC National is designed to be simple and an easy to adapt to new applications. The model consists of:

- Industries that use domestic and imported commodities (intermediate inputs) and primary factors (labour and capital) to produce goods and services. Industries may produce more than one good or service (i.e. the model allows for multi-product production).
- These goods and services are sold to other industries (as intermediate inputs into their production), or to final users (households, government, investment, stocks) or exported overseas.[1]
- There is a single category of labour that is sourced entirely from within Australia.

[1] This paper refers to household final consumption expenditure and government final consumption expenditure as demand by households and governments, respectively. It also refers to expenditure on gross fixed capital formation as investment.

- Capital services used in production are sourced from domestic and foreign owners.[2] Both domestic and foreign owners are entitled to shares of capital income earned in Australia.

PC National is a comparative-static model that assumes perfect competition in product and factor markets. It shows the difference in the economy under two different policy regimes – the simulation shows these differences between the initial and the updated structure of the economy, which is reflected in the differences between the initial and the updated database. While the model includes expenditure on investment, current investment does not increase the stock of capital available for production in the current year; it is just a category of final demand. The model does, however, allow capital to be reallocated between industries and into or out of Australia in response to the returns that prevail under the initial and the updated states.

As in similar models, changing the closure can be used to reflect different degrees of constraint in adjustments, which in turn can be interpreted as short- or long-run responses.

- Limited, short-run responses can be represented by fixing wages and capital stocks (by setting them exogenous) and allowing aggregate employment and the rate of return on capital to vary.
- Longer-run responses can be represented by fixing aggregate employment and the rate of return on capital, and allowing wages and capital stocks to vary.[3]

The model refers to three types of prices:

1. Basic prices, which reflect the cost of producing a product – for an industry, this ex-factory price is therefore calculated as the sum of all unit costs, including production taxes or subsidies.
2. Producer prices – which are calculated as the basic price plus the value of any margins required to deliver the product to the user.
3. Purchaser prices, which are producer prices plus any taxes levied on the sale of that product.

2. Database

Data sources and construction

The database for the basic PC National model is based entirely on the Australian input-output table 2018-19 (ABS 2021).[4] It contains 114 aggregate products (sourced domestically and imported), two primary factors of production (labour and capital),[5] 114 domestic producing industries and five categories of final demand (households, government, investment, inventories and exports) as well as a range of taxes (such as tariffs levied on imports, the Goods and Services Tax (GST), taxes on production, product subsidies and other taxes on products).[6]

Sets determine the size of each data item in the model database and the size of each variable in the equation system (table 1).

[2] As PC National does not include capital stocks, discussion of capital in this document focuses on the stream of capital services used in production. Sometimes capital services are referred to as capital or capital stocks for convenience.
[3] Section 4 provides more detail on capital stock closure options.
[4] It depicts the structure of the Australian economy before the beginning of Covid. At the time of writing, later databases would include characteristics that reflect the period 2020 to 2024. Future users should consider using a table that represents the economy in 2023-24 or later to avoid those characteristics.
[5] As in the input-output tables, returns to capital include gross mixed income, which implicitly includes the labour income of the self-employed. For this reason, some industries can appear more capital intensive than they might appear in other representations or statistics.
[6] Production taxes in PC National are not broken down by factor of production, as per the input-output tables.

Table 1 – Sets used in the equation system

(1) $COM(com1, \dots, com114)$: 114 commodities, indexed by *c*

(2) $IND(ind1, \dots, ind114)$: 114 industries, indexed by *i*

(3) $FAC(lab, cap)$: Two factors of production: labour and capital, indexed by *f*

(4) $SRC(dom, imp)$: Two sources of products: domestic and imported, indexed by *s*

(5) $FUSR(hou, gov, inv, stk)$: Final users: households, government, investor, and inventory changes, indexed by *u*

(6) USR: All users, industries plus final users: $= IND + FUSR$, indexed by *u*

(7) USR_stk: All users, except stk: $= USR - stk$, indexed by *u*

(8) $NCM(ncm1, \dots, ncm103)$: 103 non-margin commodities or services, indexed by *c*

(9) $MCM(mcm1, \dots, mcm11)$: 11 margin commodities or services, indexed by *m*

(10) $TX3(GST, tax, sub)$: Three taxes: GST, other product taxes and subsidies, indexed by *t*

(11) $V\_S(inv, stk)$: Two types of household saving: investment and inventory change, indexed by *s*

An aggregate version of the database can be seen in figure 1.

Figure 1 – An aggregate version of the input-output database for the model

2018-19$ million

	IND	hou	gov	inv	stk	exp	mgn	Total
COM-dom	1,466,225	716,579	362,598	351,131	-3,285	425,704	312,816	3,631,768
COM-imp	211,361	130,769	5,404	61,875	1,718			411,127
COM-mgn	103,804	155,182	3,173	16,809	184	33,663		312,816
COM-mtx	875	586	0	420	7			1,888
COM-GST	5,093	45,704	0	14,176	0	1,699		66,672
COM-tax	27,058	36,934	0	11,016	-553	0		74,456
COM-sub	-8,334	-6,249	0	-954	9	-1,234		-16,761
lab	918,895							918,895
cap	838,470							838,470
ptx	68,321							68,321
Total	3,631,768	1,079,504	371,175	454,474	-1,919	459,833	312,816	6,307,650

COM: a set of commodities; dom: domestically produced; imp: imported; IND: a set of industries; hou: households; gov: government; inv: investment; stk: inventory change; exp: exports; mgn: trade and transport margins; mtx: import duties; GST: goods and service taxes; tax: other product taxes; sub: product subsidies; lab: labour; cap: capital; ptx: production tax.

Source: Estimates based on ABS (2021).

Each section of this table represents a matrix or a vector of values. For example, the first section on the top left corner is the intermediate uses of domestically produced goods (COM) by industry (IND) valued at basic prices. It has a dimension of COM by IND.

The original ABS data set consists of 31 individual tables, only some of which are required to construct the PC National database (figure 2).[7] For example, the second row of this figure shows the basic values of imports used by industry (IND), households (hou), government (gov), investment (inv) and inventory (stk).[8] The data are sourced from ABS table 3 (Imports – supply by product group and inputs by industry and final category). The first row shows the basic values of domestically produced products used by the same categories of users and for exports and margins. These are calculated by subtracting the imports of ABS table 3 from ABS table 2, because the latter is the values of all inputs, which include domestic and imported products.

Figure 2 – Data sources of the input-output database

Reference number of the relevant ABS tables

	IND	hou	gov	inv	stk	exp	mgn
COM-dom	T2-T3	T2-T3	T2-T3	T2-T3	T2-T3	T2-T3	Sum(T23-T34)
COM-imp	T3	T3	T3	T3	T3		
COM-mgn	T23-34	T23-34	T23-34	T23-34	T23-34	T23-34	
COM-mtx	T37	T37	T37	T37	T37		
COM-GST	T36	T36	T36	T36	T36	T36	
COM-tax	T38	T38	T38	T38	T38	T38	
COM-sub	T39	T39	T39	T39	T39	T39	
lab	T2						
cap	T2						
ptx	T2						

COM: a set of commodities; dom: domestically produced; imp: imports; IND: a set of industries; hou: households; gov: government; inv: investment; stk: inventory change; exp: exports; mgn: trade and transport margins; mtx: import duties; GST: goods and service taxes; tax: other product taxes; sub: product subsidies; lab: labour; cap: capital; ptx: production tax.

Source: Estimates based on ABS (2021).

The ABS input-output tables include margin products, such as wholesale, retail and transport services. They are an important component of purchaser prices. There are originally 12 margin products. The database combines 'Pipeline' and 'Water transport' margins into a single margin product, such that the database contains only 11 margin products. The data are taken from ABS tables 23 to 34. The data in other parts of the input-output table in figure 1 can also be found in the tables indicated in figure 2.

Many of the original ABS input-output table are so-called 'product-industry' tables, in which some industries produce more than one product. As a result, the total output of an industry is not necessarily equal to the total sales of the commodity. The values of the industry outputs and the values of commodity sales are

[7] The ABS number the 31 tables from table 1 to table 39, with some gaps in the numbering (tables 11 to 16, for example, are not published). This document refers to the ABS table number.

[8] Investment in the PC National database is the sum of gross fixed capital formation by the private sector, public corporations and general government in the ABS input-output tables.

shown in table 1 of the ABS input-output data set 'Australian production by product group by industry' (ABS 2021). This table is included in the database as the MAKE matrix.[9]

Database interpretation

The ABS input-output tables provide important information on the costs of production by industry, primary distribution of income between factors of production in the Australian economy in 2018-19, and the interlinkages involved.

Additional information on the secondary distribution of national income among final users of goods and services can also be derived.

These relations can be seen more clearly in a Social Accounting Matrix (SAM).

A SAM is a representation of various national accounts, including the input-output data. Figure 3 presents the input-output data from figure 1 in a SAM framework. The rows of a SAM represent the sources of income and the columns represent the destinations of that income. In equilibrium, the sums of each row and its corresponding column must always be equal. The first row and column represent the uses and components of goods and services in purchaser prices. The second row and column present the same outputs valued at basic prices. The first two rows and columns of the SAM show the creation of national income, while the rest of the SAM shows the distribution of national income.

A SAM can be used to incorporate additional distributional information into a CGE model database, which is normally unavailable from input-output tables. This includes information on the distribution of factor incomes between their different owners, and the distribution of national income between households and government. This information is important because it determines how each economic agent balances its budget. Without collecting additional data from sources other than the input-output tables, these distributional data could be generated by assumptions.

The SAM in figure 3 shows the following assumptions in the database:

- Government expenditure is $371,175 million and total tax revenue is $194,576 million, implying that an additional tax of $176,599 million is required to balance its budget.[10] This additional tax is assumed to be imposed on all factor incomes ($1,757,365 million). It is therefore referred to in the model as 'income tax'. This implies an average 'income tax' rate of 10%.[11]
- It is assumed that people outside Australia own 30% of the national capital stock, which means that they earn 30% of all capital income (which totals $838,470 million). This means that people outside of Australia receive $251,541 million before taxation and $226,263 million after the payment of 'income tax'. This debit to the current account turns the $48,706 million trade account surplus reported in the input-output tables into a current account deficit of $177,557 million in the PC National database.

[9] PC National includes a MAKE matrix to fit the ABS input-output table as closely as possible. Many input-output tables consist of single-output industries, which simplifies the necessary code.

[10] Total tax revenue in the basic PC National database comprises: taxes on production ($68,321 million); tariff revenue ($1,888 million); GST revenue ($66,672 million); and other product tax revenue ($74,456 million); less product subsidies ($16,761 million).

[11] The assumed 'income tax' is the amount required to balance the budget. Neither the ABS input-output tables nor the model database account for all government revenues and expenditures. For example, transfers such as Newstart and various pension payments are not accounted for, nor are personal or corporate income tax. Therefore, the amount labelled as 'income tax' is not an accurate representation of income tax revenues, and the budget balance in the database is not the balance from budget papers or government financial statistics. Further, government investment is aggregated with private investment in a single investment column and is, therefore, not accounted for in the balance.

- Australian households receive all the remaining factor income of $1,505,824 million and pay 'income taxes' of $151,322 million, giving $1,354,502 million in disposable income. Of this, households allocate $1,079,504 million to consumption and $274,998 million to savings (allocating $276,917 million to investment and -$1,919 million to changes in inventories).
- Total investment is $454,474 million, which is $177,557 million more than household savings. This gap is assumed to be filled by an inflow of foreign investment. This credit in the capital and financial account offsets the debit in the current account. The external payments are balanced.

These simple assumptions could be altered with additional information about households, government or external accounts.

Figure 3 – An aggregate Social Accounting Matrix for the database

2018-19$ million

		COM	IND	lab	cap	ptx	hou
		1	2	3	4	5	6
COM	1		1,806,083				1,079,504
IND	2	3,631,768					
lab	3		918,895				
cap	4		838,470				
ptx	5		68,321				
hou	6			918,895	586,929[a]		
gov	7	126,255			25,278[a]	68,321	151,322[a]
inv	8						276,917[a]
stk	9						-1,919[a]
c/acc	10	411,127			226,263[a]		
k/acc	11						
		4,169,149	3,631,768	918,895	838,470	68,321	1,505,824

		gov	inv	stk	c/acc	k/acc	
		7	8	9	10	11	
COM	1	371,175	454,474	-1,919	459,833		4,169,149
IND	2						3,631,768
lab	3						918,895
cap	4						838,470
ptax	5						68,321
hou	6						1,505,824
gov	7						371,175
inv	8					177,557[a]	454,474
stk	9						-1,919
c/acc	10						637,390
k/acc	11				177,557[a]		177,557
		371,175	454,474	-1,919	637,390	177,557	

COM: a set of commodities; IND: a set of industries; lab: labour income; cap: capital income; ptx: production tax; hou: households; gov: government; inv: investment; stk: inventory change; c/acc: current account; k/acc: capital account.
a. The yellow shaded sections indicate data that are not directly available from the input-output table.

Source: Derived from figure 1.

3. Core equation system

The equations in PC National fall into two groups.

- Core equations and variables are essential for solving the model's general equilibrium solution.
- Non-core equations and variables are used for verification and presentation purposes. Typical non-core equations include calculations of price indices and quantity aggregates, such as macroeconomic indicators, or decomposing indicators into their contributors. These non-core equations can be added or deleted without affecting the model's solution.

The core equation system specifies the behaviours of producers and consumers. There are 27 endogenous variables and 27 equations in the core equation system.[12] The equations are organised in four sections:

i. demands for domestic, imported and exported goods (nine equations, numbers 1 to 9)
ii. final user incomes and demands for composite goods (seven equations, numbers 10 to 16)
iii. industry outputs, prices and demands for factors (six equations, numbers 17 to 22)
iv. industry supplies of multiple products (five equations, numbers 23 to 27).

The complete system of core equations can be found in appendix table A2.1.

The remainder of this section provides a brief discussion of each of the core equations.[13] The sets used in the equation system are described in table A2.1. Most equations are used to define an endogenous variable. The names of the *defined* variables match that of their defining equation names and need not be listed separately. The other endogenous variables, not defined in the system, the exogenous variables and the parameters used in the system, are described where they are first introduced.

i. Demands for domestic, imported and exported goods

The first section has nine equations (equations 1 to 9), built around two demand functions:

- *constant elasticity of substitution* (CES) demand for domestically produced and imported goods (equation 4)
- world demand for exported goods (equation 9).

The other equations define the price variables used in the two demand functions.

The first three equations define three price variables.

Equation 1 is the domestic price of imported commodity *c*, without import duty,

$$P_{(c,"imp")} = \boldsymbol{P}^{\boldsymbol{cif}}_{(c)} \boldsymbol{r}^{\boldsymbol{ex}} \qquad (c \in COM)$$

where $\boldsymbol{P}^{\boldsymbol{cif}}_{(c)}$ is the world price of imported commodity *c*, and $\boldsymbol{r}^{\boldsymbol{ex}}$ is the nominal exchange rate, defined as the foreign currency value of the domestic currency. Both are set as exogenous in the initial closure.[14]

Equation 2 defines the domestic basic price of commodity *c* from source *s* for user *u*,

$$P^{b}_{(c,u,s)} = \begin{cases} \boldsymbol{P}_{(c,s)} & (c \in COM; u \in USR; s = dom) \\ \left(1 + \boldsymbol{t}^{\boldsymbol{M}}_{(c,u)}\right) P_{(c,s)} & (c \in COM; u \in USR; s = imp) \end{cases}$$

[12] Many PC National variables are multi-dimensional and represent a group of individual variables.
[13] The system is described in levels, but the model is written in percentage changes using TABLO in the GEMPACK suite.
[14] Exogenous variables are indicated in red, undefined endogenous variables in blue, defined endogenous variables in black, and share parameters in green.

where $P_{(c,"dom")}$ is the basic price of domestically produced commodity *c*, an undefined variable, as indicated by the blue colour, and $t^{M}_{(c,u)}$ is the *ad valorem* rate of tax on imported commodity *c* for user *u*.

Equation 3 defines a *CES* price for the composite commodity *c* for user *u*, to be used in the next equation,

$$P^{b}_{\_s(c,u)} = CES\left(P^{b}_{(c,u,"dom")}, P^{b}_{(c,u,"imp")}\right) \qquad (c \in COM; u \in USR)$$

Equation 4 is the *CES* demand for domestic and imported commodities for all users, except *stk* (inventory changes), because $Q_{(c,"stk",s)}$ is assumed to be fixed exogenously.

$$Q_{(c,u,s)} = CES\left(Q_{\_s(c,u)}, P^{b}_{(c,u,"dom")}, P^{b}_{(c,u,"imp")}\right) \qquad (c \in COM; u \in USR\_stk; s \in SRC)$$

The next two equations define the purchaser price.

Equation 5 defines a producer price of commodity *c* for user *u* as its basic price plus trade and transport margins,

$$P^{t}_{\_(c,u)} = s^{D}_{(c,u)} P^{b}_{\_s(c,u)} + \sum_m s^{Dm}_{(c,u,m)} P_{(m,"dom")} \qquad (c \in COM; u \in USR)$$

where $s^{D}_{(c,u)}$ and $s^{Dm}_{(c,u,m)}$ are the share of commodity *c* and the share of margin commodity *m* used in commodity *c* for user *u*, respectively. This is the base for product taxes or subsidies that is used in the next equation to define purchaser prices.

Equation 6 defines the purchaser price as the producer price plus the commodity taxes

$$P^{t}_{(c,u)} = \left(1 + \sum_t t^{D}_{(c,u,t)}\right) P^{t}_{\_(c,u)} \qquad (c \in COM; u \in USR)$$

where $t^{D}_{(c,u,t)}$ is the *ad valorem* rate of tax or subsidy *t* on commodity *c* for user *u*.

The last three equations in this section define the world demand for Australia's exports and the world prices of the exports, used in the demand function.

Equation 7 defines the producer price of export commodity *c* as the weighted average of its basic prices of the good and the associated trade and transport margin services,

$$P^{E}_{(c)} = s^{E}_{(c)} P_{(c,"dom")} + \sum_m s^{Em}_{(c,m)} P_{(m,"dom")} \qquad (c \in COM)$$

where $s^{E}_{(c)}$ and $s^{Em}_{(c,m)}$ are the share of export commodity *c* and the share of margin commodity *m* used in the export of commodity *c*, respectively.

Equation 8 defines the world price of export commodity *c* as the sum of its producer price and any tax,

$$P^{fob}_{(c)} = \left(1 + \sum_t t^{E}_{(c,t)}\right) P^{E}_{(c)} / r^{ex} \qquad (c \in COM)$$

where $t^{E}_{(c,t)}$ is the *ad valorem* rate of tax or subsidy *t* on export commodity *c*.

The foreign demand for export commodity *c* is defined in Equation 9,

$$Q^{E}_{(c)} = \left(1 / P^{fob}_{(c)}\right)^{\sigma^{E}_{(c)}} \qquad (c \in COM)$$

where $\sigma^{E}_{(c)}$ is the elasticity of the export demand for commodity *c*.

ii. Final user incomes and demands for composite goods

The second section has seven equations (equations 10 to 16). They are centred around the demands for the composite goods by the five types of domestic users – industries, households, government, investment and inventory – shown in equation 16.

The first four equations (equations 10 to 13) define four income variables.

Equation 10 aggregates domestic and foreign factor income

$$Y^F = \sum_f \sum_i P^F_{(f,i)} Q^F_{(f,i)}$$

Equation 11 calculates foreign capital income,[15]

$$Y^{FK} = \boldsymbol{s^{FK}} \sum_i P^F_{("cap",i)} Q^F_{("cap",i)}$$

where $\boldsymbol{s^{FK}}$ is the share of foreign capital in total capital input.

Equation 12 defines household gross, or taxable, income,

$$Y^H = Y^F - Y^{FK}$$

Equation 13 calculates the aggregate rate of savings out of household income,

$$s^H_{\_s} = \sum_s s^H_{(s)}$$

where $\boldsymbol{s^H_{("inv")}}$ and $s^H_{("stk")}$ are the rates of household saving for investment and inventory change, respectively, $\boldsymbol{s^H_{("inv")}}$ is set as exogenous.[16]

Equation 14 calculates the income available for spending for four final users,

$$E_{(u)} = \begin{cases} \left(1 - \boldsymbol{s^H_{("inv")}} - s^H_{("stk")}\right)(1 - \boldsymbol{t^Y}) Y^H & (u = hou) \\ Y^G = V^{itx} + V^{etx} + V^{mtx} + V^{ptx} + V^{dtx} & (u = gov) \\ \boldsymbol{s^H_{("inv")}}(1 - \boldsymbol{t^Y}) Y^H + P^t_{\_c("inv")} \boldsymbol{Q^{FI}} & (u = inv) \\ s^H_{("stk")}(1 - \boldsymbol{t^Y}) Y^H & (u = stk) \end{cases}$$

Where $\boldsymbol{t^Y}$ is the *ad valorem* rate of 'income tax', and $\boldsymbol{Q^{FI}}$ is real foreign investment inflow. V^{itx}, V^{etx}, V^{mtx}, V^{ptx} and V^{dtx} are the revenues of taxes on household income, exports, imports, industry outputs, and domestic products, respectively, calculated in the following,

$$V^{ftx} = \boldsymbol{t^Y} Y^F$$

$$V^{etx} = \sum_c \sum_t \boldsymbol{t^E_{(c,t)}} P^E_{(c)} Q^E_{(c)}$$

$$V^{mtx} = \sum_c \sum_u \boldsymbol{t^M_{(c,u)}} P_{(c,"imp")} Q_{(c,u,"imp")}$$

$$V^{ptx} = \sum_i \boldsymbol{t^X_{(i)}} C^X_{(i)} \boldsymbol{X_{(i)}}$$

$$V^{dtx} = \sum_c \sum_u \sum_t \boldsymbol{t^D_{(c,u,t)}} P^t_{\_(c,u)} Q_{\_s(c,u)}$$

Equation 15 defines an index of purchaser prices for each user of the composite goods to be used in equation 16,

$$P^t_{\_c(u)} = \sum_c \boldsymbol{s^C_{(c,u)}} P^t_{(c,u)} \qquad (u \in IND)$$

where $\boldsymbol{s^C_{(c,u)}}$ is the value share of commodity *c* in the consumption of all composite goods by user *u*.

Equation 16 defines domestic user demands for composite goods

$$Q_{\_s(c,u)} = \begin{cases} Leontief\left(\boldsymbol{X_{(u)}}\right) & (c \in COM; u \in IND) \\ Cobb\text{-}Douglas\left(E_{(u)}, P^t_{(c,u)}\right) & (c \in COM; u = hou) \\ Leontief\left(E_{(u)} / P^t_{\_c(u)}\right) & (c \in COM; u = gov, inv) \\ \sum_s \boldsymbol{Q_{(c,u,s)}} & (c \in COM; u = stk) \end{cases}$$

There are four types of equations for five users of composite goods. An industry's demand for intermediate inputs is a *Leontief* function of its output $\boldsymbol{X_{(i)}}$, which is an endogenous variable, not defined in the system.

[15] All labour income accrues to Australian residents. Capital income accrues to both Australian residents and foreign investors.

[16] $s^H_{("stk")}$ is endogenous because changes in inventory are set to exogenous.

Household demand for composite goods is a *Cobb-Douglas* function of household income and the purchaser price of composite goods. The government and investment demand for composite goods are *Leontief* functions of real government and investment expenditure, respectively. Finally, the demand for the goods in inventory changes is determined by aggregating the fixed domestic and imported inventory goods, $\boldsymbol{Q_{(c,"stk",s)}}$.

iii. Industry outputs, prices and demands for factors

This section has six equations: five are used to define five endogenous variables (equations 17 to 21) and one is used to specify a factor market clearing condition (equation 22).

Equation 17 defines industry demand for capital and labour as a *CES* function of output and factor prices,

$$Q^F_{(f,i)} = CES\left(Q^F_{\_f(i)}, P^F_{\_f(i)}, P^F_{(f,i)}, \boldsymbol{A^F_{(f,i)}}\right) \qquad (f \in FAC; i \in IND)$$

where $\boldsymbol{A^F_{(f,i)}}$ is a factor *f* augmenting technical change variable for industry *i* and $Q^F_{\_f(i)}$ is the *Leontief* demand of industry *i* for composite factor,

$$Q^F_{\_f(i)} = Leontief\left(\boldsymbol{X_{(i)}}\right) \qquad (i \in IND)$$

and $P^F_{\_f(i)}$ is a *CES* price index, defined in the next equation.

Equation 18 defines a CES price index for composite factor in industry *i*,

$$P^F_{\_f(i)} = CES\left(P^F_{("cap",i)}, P^F_{("lab",i)}\right) \qquad (i \in IND)$$

Equation 19 defines the unit cost of an industry's output as the average costs of all inputs, weighted by their respective quantity shares,

$$C^X_{(i)} = \sum_c \boldsymbol{s^Q_{(c,i)}} P^t_{(c,i)} + \sum_f \boldsymbol{s^Q_{(f,i)}} P^F_{(f,i)} \qquad (i \in IND)$$

where $\boldsymbol{s^Q_{(c,i)}}$ and $\boldsymbol{s^Q_{(f,i)}}$ are the shares of intermediate input commodity *c* and factor *f* in the output of industry *i*, respectively, and $\sum_c \boldsymbol{s^Q_{(c,i)}} + \sum_f \boldsymbol{s^Q_{(f,i)}} = 1$.

Equation 20 defines the basic price of an industry's output as its unit cost $C^X_{(i)}$ plus a production tax,

$$P^X_{(i)} = \left(1 + \boldsymbol{t^X_{(i)}}\right) C^X_{(i)} \qquad (i \in IND)$$

where $\boldsymbol{t^X_{(i)}}$ is the ad valorem rate of production tax on the output of industry *i*.

Equation 21 defines factor prices. It stipulates that the price of a factor should be equal across all industries,

$$P^F_{(f,i)} = \boldsymbol{P^F_{\_i(f)}} \qquad (f \in FAC; i \in IND)$$

where $\boldsymbol{P^F_{\_i(f)}}$ is the basic price of factor *f*, a variable that is not defined by any equation. Equation 21 also indicates an equilibrium condition, in which homogenous factors move across industries to maximise their returns until their prices in all industries are equalised. This condition is complemented by the next equation.

Equation 22 equates the supplies of factors, $\boldsymbol{X^F_{(f)}}$, with their demands,

$$\boldsymbol{X^F_{(f)}} = \sum_i Q^F_{(f,i)} \qquad (f \in FAC)$$

Unlike previous equations, this equation does not define any endogenous variable. Instead, it stipulates a relation between variables, which occur only in equilibrium. In this case, it is a market clearing condition for capital or labour. This factor market equilibrium is achieved by autonomously adjusting the basic price of factor, $\boldsymbol{P^F_{\_i(f)}}$, because this variable is not defined in the equation system. When a factor market clears, the equilibrium value of the undefined variable, $\boldsymbol{P^F_{\_i(f)}}$, is determined simultaneously. The last two equations in this section describe the equilibrium, in which capital and labour move across industries to search for higher returns until their prices in all industries are equalised.

iv. Industry supplies of multiple products

The last section describes output supplies, including for industries that produce multiple products. This section has five equations (equations 23-27), three of which (equations 23, 25 and 27) define three endogenous variables and two of which (equations 24 and 26) specify general equilibrium conditions to determine the two remaining *undefined* variables.

Equation 23 defines the supply of commodity *c* by industry *i* as a *constant elasticity of transformation* (*CET)* function of the basic price of commodity *c*, $\boldsymbol{P}_{(c,"dom")}$, and a *CET* price index for composite output in industry *i*, $P^{C}_{(i)}$, and the output of industry *i*, $\boldsymbol{X}_{(i)}$. As, in equilibrium, this *CET* price index $P^{C}_{(i)}$ should be equal to the basic price of the industry $P^{X}_{(i)}$, equation 23 can therefore be written as a function of $P^{X}_{(i)}$,

$$X^{C}_{(c,i)} = CET\left(\boldsymbol{X}_{(i)}, \boldsymbol{P}_{(c,"dom")}, P^{X}_{(i)}\right) \qquad (c \in COM; i \in IND)$$

Equation 24 can also be expressed as,

$$P^{X}_{(i)} = CET\left(\boldsymbol{P}_{("c001","dom")}, \ldots, \boldsymbol{P}_{("c114","dom")}\right) \qquad (i \in IND)$$

Note that this equation no longer defines a price index, because $P^{X}_{(i)}$ is already defined in equation 20. Instead, equation 24 specifies a general equilibrium condition, which equates an industry's *CET* supply price index with the basic price of its outputs. This condition determines the general equilibrium value of another endogenous variable, $\boldsymbol{X}_{(i)}$, which is not defined by any equation in the system. It is also worth noting that the above *CET* supply function uses the data from the MAKE matrix in the database to identify which industries produce multiple products.[17]

Equation 25 aggregates the supply of each commodity by all its producing industries,

$$X^{C}_{\_i(c)} = \sum_i X^{C}_{(c,i)} \qquad (c \in COM)$$

This commodity supply variable is then used in equation 26 to link with its demand to specify a set of market clearing conditions,

$$X^{C}_{\_i(c)} = \begin{cases} \sum_u Q_{(c,u,"dom")} + Q^{E}_{(c)} & (c \in NCM) \\ \sum_u Q_{(c,u,"dom")} + Q^{E}_{(c)} + Q^{Mgn}_{(c)} & (c \in MCM) \end{cases}$$

These general equilibrium conditions determine the values of the last undefined variable, the basic price of commodities, $\boldsymbol{P}_{(c,"dom")}$.

Equation 27 calculates the aggregate margin demand variable $Q^{Mgn}_{(c)}$, used in equation 26,

$$Q^{Mgn}_{(m)} = \sum_c \sum_u Q^{Dm}_{(c,u,m)} + \sum_c Q^{Em}_{(c,m)} \qquad (m \in MCM)$$

where $Q^{Dm}_{(c,u,m)}$ is the *Leontief* demand for margin commodity *m* by domestic user *u* in its demand for composite commodity *c*,

$$Q^{Dm}_{(c,u,m)} = Leontief\left(Q_{\_s(c,u)}\right) \qquad (c \in COM; u \in USR; m \in MCM)$$

and $Q^{Em}_{(c,m)}$ is the *Leontief* demand for margin commodity *m* used in export commodity *c*,

$$Q^{Em}_{(c,m)} = Leontief\left(Q^{E}_{(c)}\right) \qquad (c \in COM; m \in MCM)$$

[17] The MAKE matrix details each industry's production of each commodity. The ABS Input-Output Tables refer to it as 'Australian production by product group by industry' (ABS Table 1). It is sometimes referred to as a 'supply-use' table.

Model numeraire

A CGE model is a closed system with an equal number of equations and endogenous variables. According to Walras' Law, one of the endogenous variables is redundant because its value can be derived from the values of all other variables in equilibrium. This variable can therefore be exogenously fixed, so that one equation becomes redundant. All other price changes are implicitly expressed relative to the system's numeraire. In the above system, the foreign exchange rate $\boldsymbol{r^{ex}}$ has already been chosen as the numeraire, which implies omitting one equation. The omitted equation is the external balancing condition, which stipulates that the deficit of the current account must be equal to the surplus of the capital account,[18]

$$V^{CAD} = V^{KAS}$$

where V^{CAD} is the current account deficit and V^{KAS} is the capital account surplus, defined as:

$$V^{CAD} = \sum_c \left(\boldsymbol{P^{cif}_{(c)}} \sum_u Q_{(c,u,"imp")} - P^{fob}_{(c)} Q^{E}_{(c)} \right) + (1 - \boldsymbol{t^{Y}})\, Y^{FK} / \boldsymbol{r^{ex}}$$

$$V^{KAS} = P^{t}_{\_c("inv")}\, \boldsymbol{Q^{FI}} / \boldsymbol{r^{ex}}$$

Summary of endogenous variables and equations

The core system consists of 27 endogenous variables and 27 equations. In the system, 24 equations uniquely define 24 endogenous variables. The remaining three equations specify general equilibrium conditions and determine the values of the remaining three undefined variables.

All endogenous variables and their associated equations are listed in table 2.

[18] The omitted external balance equation could be used to define a non-core variable to check the external balancing condition.

Table 2 – List of endogenous variables and equations in the core system

Eq.	Descriptions of defined variables	Defined variable	Defining variables
A.	*Demands for domestic, imported and exported goods*		
1	Domestic price of imported good *c*	$P_{(c,"imp")}$	$\boldsymbol{P^{cif}_{(c)}}$, $\boldsymbol{r^{ex}}$
2	Basic price of good *c* from source *s* for user *u*	$P^{b}_{(c,u,s)}$	$P_{(c,s)}$, $\boldsymbol{t^{M}_{(c,u)}}$
3	*CES* price index for composite good *c* for user *u*	$P^{b}_{\_s(c,u)}$	$P^{b}_{(c,u,s)}$
4	*CES* demand for composite good *c* from source *s* for user *u*	$Q_{(c,u,s)}$	$P^{b}_{(c,u,s)}$, $P^{b}_{\_s(c,u)}$, $Q_{\_s(c,u)}$
5	Producer price of good *c* for user *u*	$P^{t}_{\_(c,u)}$	$P^{b}_{\_s(c,u)}$, $P_{(c,"dom")}$
6	Purchaser price of good *c* for user *u*	$P^{t}_{(c,u)}$	$P^{t}_{\_(c,u)}$, $\boldsymbol{t^{E}_{(c,u,t)}}$
7	Producer price of exported good *c*	$P^{E}_{(c)}$	$P_{(c,"dom")}$
8	Purchaser price of exported good *c*	$P^{fob}_{(c)}$	$P^{E}_{(c)}$, $\boldsymbol{t^{E}_{(c,t)}}$, $\boldsymbol{r^{ex}}$
9	Foreign demand for exported good *c*	$Q^{E}_{(c)}$	$\boldsymbol{Y^{E}}$, $P^{fob}_{(c)}$
B.	*Final user incomes and demands for composite goods*		
10	Factor income	Y^{F}	$P^{F}_{(f,i)}$, $Q^{F}_{(f,i)}$
11	Foreign capital income	Y^{FK}	$P^{F}_{("cap",i)}$, $Q^{F}_{("cap",i)}$, $\boldsymbol{s^{FK}}$
12	Household taxable income	Y^{H}	Y^{F}, Y^{FK}
13	Household saving rates	$r^{sav}_{\_s}$	$\boldsymbol{r^{sav}_{("inv")}}$, $r^{sav}_{("stk")}$
14a	Spending income for final user *hou*	$E_{("hou")}$	$r^{sav}_{(i)}$, $\boldsymbol{t^{Y}}$, Y^{H}
14b	Spending income for final user *gov*	$E_{("gov")}$	$Y^{itx}, Y^{etx}, Y^{mtx}, Y^{ptx}, Y^{dtx}$
14c	Spending income for final user *inv*	$E_{("inv")}$	$\boldsymbol{r^{sav}_{("inv")}}$, $P^{t}_{\_c("inv")}$, Y^{H}, $\boldsymbol{Q^{FI}}$
14d	Spending income for final user *stk*	$E_{("stk")}$	$r^{sav}_{("stk")}$, $\boldsymbol{t^{Y}}$, Y^{H}
15	Purchaser price index for user *u*	$P^{t}_{\_c(u)}$	$P^{t}_{(c,u)}$
16a	*Leontief* demand of industry *i* for composite good *c*	$Q_{\_s(c,i)}$	$\boldsymbol{X_{(i)}}$
16b	*Cobb-Douglas* demand of user *hou* for composite good *c*	$Q_{\_s(c,"hou")}$	$E_{("hou")}$, $P^{t}_{(c,"hou")}$
16c	*Cobb-Douglas* demand of user *gov* for composite good *c*	$Q_{\_s(c,"gov")}$	$E_{("gov")}$, $P^{t}_{(c,"gov")}$
16d	*Leontief* demand of user *inv* for composite good *c*	$Q_{\_s(c,"inv")}$	$E_{("inv")}$, $P^{t}_{\_c("inv")}$
16e	Demand of user *stk* for composite good *c*	$Q_{\_s(c,"stk")}$	$\boldsymbol{Q_{(c,"stk",s)}}$
C.	*Industry outputs, prices and demands for factors*		
17	*CES* demand for factor *f* in industry *i*	$Q^{F}_{(f,i)}$	$Q^{F}_{\_f(i)}$, $P^{F}_{\_f(i)}$, $P^{F}_{(f,i)}$, $\boldsymbol{A^{F}_{(f,i)}}$
18	*CES* price index for composite factor in industry *i*	$P^{F}_{\_f(i)}$	$P^{F}_{(f,i)}$
19	Unit cost for output of industry *i*	$C^{X}_{(i)}$	$P^{t}_{(c,i)}$, $P^{F}_{(f,i)}$
20	Basic price of output from industry *i*	$P^{X}_{(i)}$	$C^{X}_{(i)}$, $\boldsymbol{t^{X}_{(i)}}$
21	Price of factor *f* in industry *i*	$P^{F}_{(f,i)}$	$\boldsymbol{P^{F}_{\_i(f)}}$
D.	*Industry supplies of multiple products*		
23	*CET* supply of good *c* by industry *i*	$X^{c}_{(c,i)}$	$\boldsymbol{X_{(i)}}$, $\boldsymbol{P_{(c,"dom")}}$, $P^{X}_{(i)}$
25	Supply of good *c* from all industries	$X^{c}_{\_i(c)}$	$X^{c}_{(c,i)}$
27	Total demand for margin good *m*	$Q^{Mgn}_{(m)}$	$Q_{\_s(c,u)}$, $Q^{E}_{(c)}$

Eq.	Descriptions of equilibrium conditions	Undefined variables	Descriptions of undefined variables
22	Market clearing condition factor for factor *f*	$P^{F}_{\_i(f)}$	Basic price of factor *f*
24	Linking $P^{X}_{(i)}$ with *CET* price index	$X_{(i)}$	Output of industry *i*
26	Market clearing condition for good *c*	$P_{(c,"dom")}$	Basic price of domestic good *c*

Light green shading: demand functions. Blue shading and notation: equilibrium conditions for undefined variables.

In the core system, each endogenous variable has its own corresponding equation. As a result, the number of endogenous variables naturally matches the number of equations, implying a unique solution for the core system. When expressed as percentage changes, the system becomes linear and can be solved using matrix algebra.

Table 2 shows that the entire model is built around four sets of demand functions (shaded by light green):

- the domestic demand for goods, $Q_{(c,u,s)}$
- the domestic demand for composites goods, $Q_{\_s(c,u)}$
- the foreign demand for exports, $Q^{E}_{(c)}$
- the industry demand for factors, $Q^{F}_{(f,i)}$.

All the other variables calculate the price or income terms used in the above demand functions. The model assumes constant returns to scale (CRTS) technology in domestic production. As a result, the outputs of industries are determined by their demand and the product prices reflect the cost of production. For given supplies of factors, the above demand functions therefore determine the general equilibrium solution.[19]

The variables in the core system can also be used to show how they are related to the values in the database as illustrated in figure 4. The initial equilibrium allows model variables to reproduce all the values in the database.

[19] A more detailed discussion on the role of CRTS in solving CGE models can be found in Zhang (2023).

Figure 4 – Derivation of input-output tables

Derivation of the basic price table

	IND	hou	gov	inv	stk	exp	mgn	Total
COM-dom	$P_{(c,"dom")}Q_{(c,u,"dom")}$					$P_{(c,"dom")}Q^{E}_{(c)}$	$P_{(m,"dom")}Q^{Mgn}_{(m)}$	$P_{(c,"dom")}X^{C}_{\_i(c)}$
COM-imp	$(1+t^{M}_{(c,u)})P_{(c,"imp")}Q_{(c,u,"imp")}$							
COM-mgn	$P_{(m,"dom")}Q^{Dm}_{(c,u,m)}$					$P_{(m,"dom")}Q^{Em}_{(c,m)}$		
COM-mtx	$t^{M}_{(c,u)}P_{(c,"imp")}Q_{(c,u,"imp")}$							
COM-GST	$t^{D}_{(c,u"GST")}P^{t}_{\_(c,u)}Q_{\_s(c,u)}$					$t^{E}_{(c,"GST")}P^{E}_{(c)}Q^{E}_{(c)}$		
COM-tax	$t^{D}_{(c,u"tax")}P^{t}_{\_(c,u)}Q_{\_s(c,u)}$					$t^{E}_{(c,"tax")}P^{E}_{(c)}Q^{E}_{(c)}$		
COM-sub	$t^{D}_{(c,u"sub")}P^{t}_{\_(c,u)}Q_{\_s(c,u)}$					$t^{E}_{(c,"sub")}P^{E}_{(c)}Q^{E}_{(c)}$		
lab	$P^{F}_{("lab",i)}Q^{F}_{("lab",i)}$							
cap	$P^{F}_{("cap",i)}Q^{F}_{("cap",i)}$							
ptx	$t^{X}_{(i)}C^{X}_{(i)}X_{(i)}$							
Total	$P^{X}_{(i)}X_{(i)}$							

Derivation of the purchaser price table

	IND	hou	gov	inv	stk	exp
COM-composite	$P^{t}_{(c,u)}Q_{\_s(c,u)}$					$r^{ex}P^{fob}_{(c)}Q^{E}_{(c)}$
FAC-composite	$P^{F}_{\_f(i)}Q^{F}_{\_f(i)}$					
ptx	$t^{X}_{(i)}C^{X}_{(i)}X_{(i)}$					
Total	$P^{X}_{(i)}X_{(i)}$					

4. Macro closure

In CGE models, simulation results depend on how the modelled economy and its agents respond to a policy change. These responses are conditioned by the economic environment in which they operate, or the so-called macro closure, as well as the functional forms and model parameterisation. The setting of such an environment depends on the purpose of the policy simulation.

Closure options

In the initial closure above, the supply of domestic capital is fixed. This can be interpreted as representing a short-run equilibrium condition.

Alternative macro closures are possible.

Three alternative closure options are set out below and can be interpreted as long-run closures, that is, the new equilibrium accounts for adjustments in the allocation of global capital.

1. This option assumes that the Australian rate of return to capital is determined by the world rate of return. When a policy change increases the domestic rate of return above the world level, foreign capital moves in until the Australian rate of return returns to the world level. This can be implemented by introducing a new endogenous variable for the rate of return, defined as the rental price of capital divided by the purchaser price of investment,
   $R^{cap} = P^{F}_{\_i("cap")}/P^{t}_{\_c("inv")}$
   and setting R^{cap} exogenous and the share of foreign capital s^{FK} endogenous;

2. This option assumes that any foreign capital inflow is driven by an increase in the domestic rental price of capital $P^{F}_{\_i("cap")}$. This is consistent with assuming that the rental price capital is determined by the world market. Any foreign capital inflow reduces the domestic rental price to the world level. This option can be implemented by introducing a new exogenous variable for the world rental price of capital, $\boldsymbol{P^{FK}}$ and linking it with the domestic rental price,
$$\boldsymbol{P^{FK}} = P^{F}_{\_i("cap")} / \boldsymbol{r^{ex}}$$
3. This option assumes imperfect international capital mobility. It implies that an inflow of foreign capital reduces the domestic rental price or the rate of return to capital, but not to the world level. In this case, a supply function of foreign capital may be introduced as follows,
$$\boldsymbol{s^{FK}} = \left(P^{FK}\right)^{\boldsymbol{\sigma^{FK}}}$$
where the foreign rental price of capital, P^{FK}, is set as endogenous. Foreign capital mobility is now controlled by the value of the supply elasticity $\boldsymbol{\sigma^{FK}}$, which can be set between zero and infinity. If set at zero, there is no foreign capital inflow. If infinitely large, capital becomes perfectly mobile, implying an unlimited supply of capital at a constant rental price. With intermediate values, capital becomes imperfectly mobile, and the domestic rental price of capital may remain above the world price in equilibrium.

Foreign capital inflow should be linked with foreign investment. This is because an increase in capital stock normally results from an accumulation of investment. Therefore, an increase in foreign capital stock should be associated with an increase in foreign investment. This can be implemented by introducing the following equation, which links net foreign investment with the foreign capital share,
$$Q^{FI} = \boldsymbol{s^{FK}}$$
where the original exogenous variable $\boldsymbol{Q^{FI}}$ becomes an endogenous variable, determined by the foreign capital share. A rise in foreign investment should have little impact on domestic production, because it only shifts the demand from exports to domestic investment and consumption. However, an increase in foreign investment should help reduce a country's current account deficit, ease a downward pressure on export prices and, therefore, improve its terms of trade.

5. Other non-core variables

In addition to the core variables, other variables can be introduced to process the results from a simulation for presentation purposes. Among these variables, some commonly used national aggregates are defined in this section.

Measuring national aggregates

Three key national indicators can be calculated from the simulation results:

- real gross domestic product (GDP)
- real gross national product (GNP)
- real gross national absorption (GNA).

The first is a measure of domestic production activity. The second is a measure of real national income, that accrues to Australians at home and abroad – it excludes returns to foreign-owned capital used in domestic production. The third is a measure of real national expenditure or the purchasing power of nominal national income (GNP). While real GNP is a supply-side measure of national income, real GNA is a demand-side measure of national income. It is, therefore, related to a measure of welfare, because it reflects the real benefits that a country's residents can obtain from the income they receive (Zhang 2019).

Table 3 shows how the above three national aggregates are defined in the model. These three indicators are related. GDP is an aggregate of all the products or income, produced in the country. GNP measures only the part of the GDP that accrues to its own residents. Therefore, it can be defined as GDP net of foreign capital income in the model. In nominal terms, GNA is equal to GNP, because they both are measures of national income. In real terms, however, they are generally not equal. This is because GNP is measured on the supply side and deflated with supply prices, while GNA is measured on the demand side and deflated with demand prices. The GNP deflator is unlikely to be the same as the GNA deflator, because the underlying commodity compositions of the two side measurements are quite different. The measure of real GNA is defined as nominal GNP, deflated by a GNA deflator, which includes the influences of changes in domestic prices and changes in prices in the components of the current and capital accounts. The most important influence is the changes in the world prices of exports, relative to that of imports, or the terms of trade, which usually dominate the difference between real GNP and GNA.[20]

Table 3 – Explanatory variables for national aggregates

(1) Gross Domestic Product (GDP), measured on the income side

$$V^{GDP}_{("inc")} = Y^F + V^{ptx} + V^{dtx} + V^{etx} + V^{mtx}$$

(2) Gross Domestic Product (GDP), measured on the expenditure side

$$V^{GDP}_{("exp")} = \sum_c \sum_{u \in FUSR} \left(P^t_{(c,u)} Q_{\_s(c,u)} - P^b_{(c,u,"imp")} Q_{(c,u,"imp")}\right) + \sum_c \left(1 + \sum_t \boldsymbol{t}^{\boldsymbol{E}}_{\boldsymbol{(c,t)}}\right) P^E_{(c)} Q^E_{(c)}$$

(3) Gross National Product (GNP), measured on the income side

$$V^{GNP} = V^{GDP}_{("inc")} - (1 - \boldsymbol{t}^{\boldsymbol{Y}}) \boldsymbol{s}^{\boldsymbol{FK}} \sum_i P^F_{("cap",i)} Q^F_{("cap",i)}$$

(4) Real Gross Domestic Product (GDP): a measure of real output produced in the country

$$Q^{GDP} = V^{GDP} / P^{GDP}$$

where P^{GDP} is GDP deflator,

$$P^{GDP} = \sum_c \sum_{u \in FUSR} \left(\boldsymbol{s}^{\boldsymbol{q}}_{\boldsymbol{(c,u)}} P^t_{(c,u)} - \boldsymbol{s}^{\boldsymbol{qm}}_{\boldsymbol{(c,u)}} P^b_{(c,u,"imp")}\right) + \sum_c \boldsymbol{s}^{\boldsymbol{qE}}_{\boldsymbol{(c)}} \left(1 + \sum_t \boldsymbol{t}^{\boldsymbol{E}}_{\boldsymbol{(c,t)}}\right) P^E_{(c)}$$

where $\boldsymbol{s}^{\boldsymbol{q}}_{\boldsymbol{(c,u)}}$, $\boldsymbol{s}^{\boldsymbol{qm}}_{\boldsymbol{(c,u)}}$ and $\boldsymbol{s}^{\boldsymbol{qE}}_{\boldsymbol{(c)}}$ are the quantity shares of composite good *c* and imported good *c* for final user *u*, and for the exported good *c* in real GDP, respectively.

(5) Real Gross National Product (GNP): a measure of real national income

$$Q^{GNP} = V^{GNP} / P^{GNP}$$

where P^{GNP} is GNP deflator,

$$P^{GNP} = \boldsymbol{s}^{\boldsymbol{GDP}} P^{GDP} - \boldsymbol{s}^{\boldsymbol{FKI}} P^F_{\_i("cap")}$$

where $\boldsymbol{s}^{\boldsymbol{GDP}}$ and $\boldsymbol{s}^{\boldsymbol{FKI}}$ are the shares of real GDP and real foreign capital income in real GNP, respectively, $\boldsymbol{s}^{\boldsymbol{GDP}} + \boldsymbol{s}^{\boldsymbol{FKI}} = 1$.

(6) Real Gross National Absorption (GNA): a measure of real national expenditure

$$Q^{GNA} = V^{GNP} / P^{GNA}$$

where P^{GNA} is GNA deflator,

$$P^{GNA} = \boldsymbol{s}^{\boldsymbol{GNP}} P^{GNP} + \boldsymbol{s}^{\boldsymbol{CAB}} P^{CAB} - \boldsymbol{s}^{\boldsymbol{KAB}} P^{KAB}$$

where $\boldsymbol{s}^{\boldsymbol{GNP}}$, $\boldsymbol{s}^{\boldsymbol{CAB}}$ and $\boldsymbol{s}^{\boldsymbol{KAB}}$ are the shares of real GNP, current account balance and capital account balance in real GNA, respectively.

[20] For a more detailed introduction to GNA, see Zhang (2019).

Appendix 1: Industries in the PC National database

Table A.1 – Industries in the PC National database[a]

Number	Industry	Number	Industry
1	Sheep, grains, beef and dairy cattle	31	Footwear manufacturing
2	Poultry and other livestock	32	Sawmill product manufacturing
3	Other agriculture	33	Other wood product manufacturing
4	Aquaculture	34	Pulp, paper and paperboard manufacturing
5	Forestry and logging	35	Paper stationery and other converted paper product manufacturing
6	Fishing, hunting and trapping	36	Printing (including the reproduction of recorded media)
7	Agriculture, forestry and fishing support services	37	Petroleum and coal product manufacturing
8	Coal mining	38	Human pharmaceutical and medicinal product manufacturing
9	Oil and gas extraction	39	Veterinary pharmaceutical and medicinal product manufacturing
10	Iron ore mining	40	Basic chemical manufacturing
11	Non-ferrous metal ore mining	41	Cleaning compounds and toiletry preparation manufacturing
12	Non-metallic mineral mining	42	Polymer product manufacturing
13	Exploration and mining support services	43	Natural rubber product manufacturing
14	Meat and meat product manufacturing	44	Glass and glass product manufacturing
15	Processed seafood manufacturing	45	Ceramic product manufacturing
16	Dairy product manufacturing	46	Cement, lime and ready-mixed concrete manufacturing
17	Fruit and vegetable product manufacturing	47	Plaster and concrete product manufacturing
18	Oils and fats manufacturing	48	Other non-metallic mineral product manufacturing
19	Grain mill and cereal product manufacturing	49	Iron and steel manufacturing
20	Bakery product manufacturing	50	Basic non-ferrous metal manufacturing
21	Sugar and confectionery manufacturing	51	Forged iron and steel product manufacturing
22	Other food product manufacturing	52	Structural metal product manufacturing
23	Soft drinks, cordials and syrup manufacturing	53	Metal containers and other sheet metal product manufacturing
24	Beer manufacturing	54	Other fabricated metal product manufacturing
25	Wine, spirits and tobacco	55	Motor vehicles and parts; other transport equipment manufacturing
26	Textile manufacturing	56	Ships and boat manufacturing
27	Tanned leather, dressed fur and leather product manufacturing	57	Railway rolling stock manufacturing
28	Textile product manufacturing	58	Aircraft manufacturing
29	Knitted product manufacturing	59	Professional, scientific, computer and electronic equipment manufacturing
30	Clothing manufacturing	60	Electrical equipment manufacturing

Number	Industry	Number	Industry
61	Domestic appliance manufacturing	88	Telecommunication services
62	Specialised and other machinery and equipment manufacturing	89	Library and other information services
63	Furniture manufacturing	90	Finance
64	Other manufactured products	91	Insurance and superannuation funds
65	Electricity generation	92	Auxiliary finance and insurance services
66	Electricity transmission, distribution, on selling and electricity market operation	93	Rental and hiring services (except real estate)
67	Gas supply	94	Ownership of dwellings
68	Water supply, sewerage and drainage services	95	Non-residential property operators and real estate services
69	Waste collection, treatment and disposal services	96	Professional, scientific and technical services
70	Residential building construction	97	Computer systems design and related services
71	Non-residential building construction	98	Employment, travel agency and other administrative services
72	Heavy and civil engineering construction	99	Building cleaning, pest control and other support services
73	Construction services	100	Public administration and regulatory services
74	Wholesale trade	101	Defence
75	Retail trade	102	Public order and safety
76	Accommodation	103	Primary and secondary education services (incl pre-schools and special schools)
77	Food and beverage services	104	Technical, vocational and tertiary education services (including undergraduate and postgraduate)
78	Road transport	105	Arts, sports, adult and other education services (including community education)
79	Rail transport	106	Health care services
80	Water, pipeline and other transport	107	Residential care and social assistance services
81	Air and space transport	108	Heritage, creative and performing arts
82	Postal and courier pick-up and delivery service	109	Sports and recreation
83	Transport support services and storage	110	Gambling
84	Publishing (except internet and music publishing)	111	Automotive repair and maintenance
85	Motion picture and sound recording	112	Other repair and maintenance
86	Broadcasting (except internet)	113	Personal services
87	Internet service providers, internet publishing and broadcasting, websearch portals and data processing	114	Other services

a. Industries 1 to 7 form the agricultural sector, 8 to 13 the mining sector, 14 to 64 the manufacturing sector, and 65 to 114 the services sector.

Appendix 2: Core equation system

Table A.2 – Core equations of the PC National model: the basic version

Sets used in the equation system

1. Sets

(1) COM: 114 commodities (*com1*, …, *com114*);
(2) IND: 114 industries (*ind1*, …, *ind114*);
(3) FAC: factors (*cap*, *lab*);
(4) SRC: two sources of products: domestically produced and imported (*dom*, *imp*);
(5) $FUSR$: Final users: households, government, investor and inventory changes (*hou*, *gov*, *inv*, *stk*);
(6) USR: All users: $= IND + FUSR$;
(7) USR_stk: All users except *stk*: $= USR - stk$;
(8) NCM: 103 non-margin commodities or services (*ncm1*, …, *ncm103*);
(9) MCM: 11 margin commodities or services (*mcm1*, …, *mcm11*);
(10) $TX3$: 3 taxes: GST, other taxes, and subsidies (*GST*, *tax*, *sub*);
(11) V_S: two types of household saving: investment and inventory change.

Core equations for endogenous variables (1–28)

1. Demands for domestic, imported and exported goods (9 equations, numbers 1 to 9)

(1) Domestic price of import COM *c*, tariff excluded

$$P_{(c,"imp")} = \boldsymbol{P}^{\boldsymbol{cif}}_{\boldsymbol{(c)}} \boldsymbol{r}^{\boldsymbol{ex}} \qquad (c \in COM)$$

where $\boldsymbol{P}^{\boldsymbol{cif}}_{\boldsymbol{(c)}}$ is the world price of import COM *c*, and $\boldsymbol{r}^{\boldsymbol{ex}}$ is the exchange rate, the foreign currency value of a unit of domestic currency, set as exogenous.

(2) Domestic basic price of COM *c* from SRC *s* for USR *u*,

$$P^{b}_{(c,u,s)} = \begin{cases} \boldsymbol{P}_{\boldsymbol{(c,s)}} & (c \in COM; u \in USR; s = dom) \\ \left(1 + \boldsymbol{t}^{\boldsymbol{M}}_{\boldsymbol{(c,u)}}\right) P_{(c,s)} & (c \in COM; u \in USR; s = imp) \end{cases}$$

where $\boldsymbol{P}_{\boldsymbol{(c,"dom")}}$ is the basic price of domestically produced COM *c*, an undefined variable, and $\boldsymbol{t}^{\boldsymbol{M}}_{\boldsymbol{(c,u)}}$ is the *ad valorem* rate of tax on import COM *c* for USR *u*.

(3) CES basic price of composite COM *c* for USR *u*,

$$P^{b}_{\_s(c,u)} = CES\left(P^{b}_{(c,u,"dom")}, P^{b}_{(c,u,"imp")}\right) \qquad (c \in COM; u \in USR)$$

(4) *CES* demand for COM *c* from SRC *s* for USR *u*

$$Q_{(c,u,s)} = CES\left(Q_{\_s(c,u)}, P^{b}_{(c,u,"dom")}, P^{b}_{(c,u,"imp")}\right) \qquad (c \in COM; u \in USR\_stk; s \in SRC)$$

(5) Producer price of COM *c* for USR *u*,

$$P^{t}_{\_(c,u)} = \boldsymbol{s}^{\boldsymbol{D}}_{\boldsymbol{(c,u)}} P^{b}_{\_s(c,u)} + \sum_m \boldsymbol{s}^{\boldsymbol{Dm}}_{\boldsymbol{(c,u,m)}} P_{(m,"dom")} \qquad (c \in COM; u \in USR)$$

where $\boldsymbol{s}^{\boldsymbol{D}}_{\boldsymbol{(c,u)}}$ and $\boldsymbol{s}^{\boldsymbol{Dm}}_{\boldsymbol{(c,u,m)}}$ are the share of COM *c* and the share of MCM *m* in the demand for COM *c*, respectively.

(6) Purchaser price of COM *c* for USR *u*

$$P^{t}_{(c,u)} = \left(1 + \sum_t \boldsymbol{t}^{\boldsymbol{D}}_{\boldsymbol{(c,u,t)}}\right) P^{t}_{\_(c,u)} \qquad (c \in COM; u \in USR)$$

where $\boldsymbol{t}^{\boldsymbol{D}}_{\boldsymbol{(c,u,t)}}$ is the *ad valorem* rate of tax *t* on COM *c* for USR *u*.

(7) Producer price of export com *c*

$$P^{E}_{(c)} = \boldsymbol{s}^{\boldsymbol{E}}_{\boldsymbol{(c)}} \boldsymbol{P}_{\boldsymbol{(c,"dom")}} + \sum_m \boldsymbol{s}^{\boldsymbol{Em}}_{\boldsymbol{(c,m)}} \boldsymbol{P}_{\boldsymbol{(m,"dom")}} \qquad (c \in COM)$$

where $s^{E}_{(c)}$ and $s^{Em}_{(c,m)}$ are the shares of COM *c* and the share of MCM *m* in the demand for export COM *c*, respectively and $s^{E}_{(c)} + \sum_m s^{Em}_{(c,m)} = 1$ for export COM *c*.

(8) World purchaser price of export COM *c*

$$P^{fob}_{(c)} = \left(1 + \sum_t t^{E}_{(c,t)}\right) P^{E}_{(c)} / r^{ex} \qquad (c \in COM)$$

where $t^{E}_{(c,t)}$ is the *ad valorem* rate of tax *t* on export COM *c*.

(9) Foreign demand for export COM *c*

$$Q^{E}_{(c)} = \left(1/P^{fob}_{(c)}\right)^{\sigma^{E}_{(c)}} \qquad (c \in COM)$$

where $\sigma^{E}_{(c)}$ is the elasticity of demand for COM *c*.

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2. Final user incomes and demands for composite goods (7 equations, numbers 10 to 16)

(10) Factor income

$$Y^{F} = \sum_f \sum_i P^{F}_{(f,i)} Q^{F}_{(f,i)}$$

(11) Foreign capital income

$$Y^{FK} = s^{FK} \sum_i P^{F}_{("cap",i)} Q^{F}_{("cap",i)}$$

where s^{FK} is the share of foreign capital in total capital input.

(12) Household taxable income

$$Y^{H} = Y^{F} - Y^{FK}$$

(13) Household aggregate saving rate

$$s^{H}_{\_s} = \sum_s s^{H}_{(s)}$$

where $s^{H}_{(s)}$ is the rate of household saving for investment or inventory changes: $s^{H}_{("inv")}$ is set as exogenous.

(14) Final USR *u* expenditure income

$$E_{(u)} = \begin{cases} \left(1 - s^{H}_{("inv")} - s^{H}_{("stk")}\right)(1 - t^{Y}) Y^{H} & (u = hou) \\ V^{itx} + V^{etx} + V^{mtx} + V^{ptx} + V^{dtx} & (u = gov) \\ s^{H}_{("inv")} (1 - t^{Y}) Y^{H} + P^{t}_{\_c(\text{inv})} Q^{FI} & (u = inv) \\ s^{H}_{("stk")} (1 - t^{Y}) Y^{H} & (u = stk) \end{cases}$$

where t^{Y} is the *ad valorem* rate of 'income tax' and Q^{FI} is real foreign investment inflow. Government revenue V^{itx}, V^{etx}, V^{mtx}, V^{dtx} and V^{ptx} are the taxes on factor incomes, traded and domestically produced commodities, and on the outputs of industries, defined as,

$$V^{ftx} = t^{Y} Y^{F}$$

$$V^{etx} = \sum_c \sum_t t^{E}_{(c,t)} P^{E}_{(c)} Q^{E}_{(c)}$$

$$V^{mtx} = \sum_c \sum_u t^{M}_{(c,u)} P_{(c,"imp")} Q_{(c,u,"imp")}$$

$$V^{dtx} = \sum_c \sum_u \sum_t t^{D}_{(c,u,t)} P^{t}_{\_(c,u)} Q_{\_s(c,u)}$$

$$V^{ptx} = \sum_i t^{X}_{(i)} C^{X}_{(i)} X_{(i)}$$

(15) Purchaser price index for USR *u*

$$P^{t}_{\_c(u)} = \sum_c s^{C}_{(c,u)} P^{t}_{(c,u)} \qquad (u \in USR)$$

where $s^{C}_{(c,u)}$ is the share of COM *c* in the consumption of USR *u*.

(16) Demand of USR *u* for composite COM *c*

$$Q_{\_s(c,u)} = \begin{cases} Leontief\left(X_{(u)}\right) & (c \in COM; u \in IND) \\ Cobb\text{-}Douglas\left(E_{(u)}, P^{t}_{(c,u)}\right) & (c \in COM; u = hou) \\ Leontief\left(E_{(u)} / P^{t}_{\_c(u)}\right) & (c \in COM; u = gov, inv) \\ \sum_s Q_{(c,u,s)} & (c \in COM; u = stk) \end{cases}$$

Note that $X_{(i)}$ is the output of IND *i*, an undefined variable, and $Q_{(c,"stk",s)}$ is set as exogenous.

3. Industry outputs, prices and demands for factors (6 equations: numbers 17 to 22)

(17) *CES* demand of IND *i* for FAC *f*

$$Q^F_{(f,i)} = CES\left(Q^F_{\_f(i)}, P^F_{\_f(i)}, P^F_{(f,i)}, A^F_{(f,i)}\right) \qquad (f \in FAC; i \in IND)$$

where $A^F_{(f,i)}$ is a FAC *f* augmenting technical change variable for IND *i* and $Q^F_{\_f(i)}$ is the *Leontief* demand for composite factor in IND *i*,

$$Q^F_{\_f(i)} = Leontief\left(X_{(i)}\right) \qquad (i \in IND)$$

(18) *CES* price index for composite factor in IND *i*

$$P^F_{\_f(i)} = CES\left(P^F_{("cap",i)}, P^F_{("lab",i)}\right) \qquad (i \in IND)$$

(19) Unit cost of output of IND *i* (Zero pure profit condition)

$$C^X_{(i)} = \sum_c s^Q_{(c,i)} P^t_{(c,i)} + \sum_f s^Q_{(f,i)} P^F_{(f,i)} \qquad (i \in IND)$$

where $s^Q_{(c,i)}$ and $s^Q_{(f,i)}$ are the shares of intermediate input COM *c* and FAC *f* in the output of IND *i*, respectively, and $\sum_c s^Q_{(c,i)} + \sum_f s^Q_{(f,i)} = 1$ for IND *i*.

(20) Basic price for the output of IND *i*

$$P^X_{(i)} = \left(1 + t^X_{(i)}\right) C^X_{(i)} \qquad (i \in IND)$$

where $t^X_{(i)}$ is the rate of production tax on the output of IND *i*.

(21) Factor price equalisation in equilibrium

$$P^F_{(f,i)} = P^F_{\_i(f)} \qquad (f \in FAC; i \in IND)$$

where $P^F_{\_i(f)}$ is the basic price of FAC *f*, an undefined variable.

(22) Market equilibrium condition for FAC *f* to determine $P^F_{\_i(f)}$

$$X^F_{(f)} = \sum_i Q^F_{(f,i)} \qquad (f \in FAC)$$

where $X^F_{(f)}$ is the supply of FAC *f*, an exogenous variable.

4. Industry supplies of multiple products (6 equations: numbers 23 to 28)

(23) *CET* supply of COM *c* by IND *i*

$$X^C_{(c,i)} = CET\left(X_{(i)}, P_{(c,"dom")}, P^X_{(i)}\right) \qquad (c \in COM; i \in IND)$$

(24) *CET* price index for composite good from IND *i* to determine $X_{(i)}$

$$P^X_{(i)} = CET\left(P_{("c001","dom")}, \ldots, P_{("c114","dom")}\right) \qquad (i \in IND)$$

Note that this equation links the basic price $P^X_{(i)}$ with a *CET* price index, which is an equilibrium condition to determine the value of undefined variable $X_{(i)}$.

(25) Total supply of COM *c*

$$X^C_{\_i(c)} = \sum_i X^C_{(c,i)} \qquad (c \in COM)$$

(26) Total supply of COM *c* equal to total sale of COM *c* to determine $P_{(c,"dom")}$

$$X^C_{\_i(c)} = \begin{cases} \sum_u Q_{(c,u,"dom")} + Q^E_{(c)} & (c \in NCM) \\ \sum_u Q_{(c,u,"dom")} + Q^E_{(c)} + Q^{Mgn}_{(c)} & (c \in MCM) \end{cases}$$

(27) Total demand for margin MCM *m*

$$Q^{Mgn}_{(m)} = \sum_c \sum_u Q^{Dm}_{(c,u,m)} + \sum_c Q^{Em}_{(c,m)} \qquad (m \in MCM)$$

where $Q^{Dm}_{(c,u,m)}$ is the *Leontief* demand for margin MCM *m* by domestic USR *u* in its demand for composite COM *c*,

$$Q^{Dm}_{(c,u,m)} = Leontief\left(Q_{\_s(c,u)}\right) \qquad (c \in COM; u \in USR; m \in MCM)$$

and $Q^{Em}_{(c,m)}$ is the *Leontief* demand for margin MCM *m* used in export COM *c*,

$$Q^{Em}_{(c,m)} = Leontief\left(Q^{E}_{(c)}\right) \qquad (c \in COM; m \in MCM)$$

1. Variables in red are set as exogenous in the basic closure.
2. Variables in blue are *undefined* and need to be determined by their corresponding *market clearing conditions*.
3. Green colour is used for parameters, such as various shares and elasticities.
4. Equations shaded in blue are the *general equilibrium conditions* required to determine the *undefined* variables.

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